THE KINETIC THEORY OF GASES

A Treatise on the Kinetic Theory of Gases. By Henry William Watson, M.A., formerly Fellow of Trinity College, Cambridge. (Oxford: Clarendon Press, 1876.)

THIS book does not profess to treat of all that has been written about the kinetic theory of gases. It discusses the ultimate average condition of a material system, consisting of a very great number of parts in motion within a confined space, and it follows for the most part the methods of investigation given by Boltzmann. The discussion is arranged in the form of thirteen propositions, in which the different cases are considered in the order of their complexity. In the earlier propositions the moving bodies are supposed to be rigid-elastic spheres acting on each other only by impact, afterwards external forces are introduced, and finally the bodies are supposed to be material systems, the parts of which are held together by any system of forces consistent with the principle of the conservation of energy.

The ultimate average condition of such a system is investigated in a very satisfactory manner in this book. No part of mathematical science requires more careful handling than that which treats of probabilities and averages. Mathematicians, whose competence to deal with other questions is undoubted, have fallen into errors in treating of probabilities, and even the validity of certain methods of proof is still apparently an open question.

Besides this, some of the consequences to which these theorems lead us are so startling that we are not prepared to admit them without an unanswerable proof, and of the investigations already given, some are so short and incomplete, and others so long and roundabout, that it requires no ordinary exercise both of penetration and of patience to find out whether they are proofs at all. Mr. Watson has conferred a great benefit on the students of the kinetic theory by placing before them in a series of distinct propositions, none of them too long for the mind to grasp, all the necessary steps leading to the result, and none of the superfluous evolutions in which the mental energy of the student is so often dissipated. The book, as we have said before, is confined to the investigation of the ultimate average condition of the system, and does not discuss the processes of diffusion by which that ultimate condition is attained, such as the inter-diffusion of gases, the diffusion of momentum by viscosity, and the diffusion of energy by thermal conduction. These have been recently treated in a larger work,[1] to which we may have occasion to refer.

There are two very different methods of defining and investigating the state of a complex material system. According to the strict dynamical method the particles of the system are defined in any sufficient manner, as, for instance, by their co-ordinates at a given epoch, and the position of any particle at any other time is then defined by its co-ordinates, expressed as functions of the time, the form of these functions being different from particle to particle, and not necessarily continuous in passing from one particle to another which was contiguous to it in the initial configuration.

According to this method our analysis enables us to trace every particle throughout its whole course, and therefore we can apply the laws of motion in all their strictness.

The application of this method to systems consisting of large numbers of bodies is out of the question. We therefore make use of another method which we may call the statistical method, on account of its analogy with the methods employed in dealing with the fluctuations of a large population.

We divide the bodies of the system into groups according to their position, their velocity, or any other property belonging to them, and we fix our attention not on the bodies themselves, but on the *number* belonging at any instant to one particular group. This number is, of course, subject to change on account of bodies entering or leaving the group, and we have therefore to study the conditions under which bodies enter or leave the group, and in so doing we must follow the course of the bodies according to the dynamical method. But as soon as the process is over, when the body has fairly entered the group or left it, we withdraw our attention from the body, and if it should come before us again we treat it as a new body, just as the turnstile at an exhibition counts the visitors who enter without respect to what they have done or are going to do, or whether they have passed through the turnstile before.

The first mode of grouping the bodies of the system is to class those together which, at a given time, are in a given region of space. This is called grouping according to configuration, and what we learn from it is the distribution of the positions or co-ordinates of the bodies in space.

The second mode of grouping is that according to velocity. The best way to understand this is to suppose a diagram of velocities constructed by drawing from a given point as origin a system of vectors representing in direction and magnitude the velocities of the different bodies. The extremities of these vectors are called the velocity-points of the bodies to which they correspond, and by grouping the bodies according to the regions of the diagram in which their velocity-points lie, we learn from the numbers in the groups the distribution of velocities among the bodies.

In like manner we may form groups defined in any other way, as, for instance, those pairs of bodies whose distance from one another lies between given limits, and by confining within sufficiently narrow limits the values of all the properties of the bodies which form the group, we may consider all the bodies belonging to the group as practically in the same state. Whether at a given instant any body actually belongs to the group is, of course, another question.

The object of study in the statistical method is the probable number of bodies in each group. We may get rid of the idea of probability by supposing the system to continue under the same conditions for a very long time. During this time many bodies will enter the group, stay in it for a certain time, and then leave it. If we add together the times of residence within the group of all these bodies, and divide the sum by the whole time of observation, we obtain a numerical quantity which we may call the average number of bodies in the group. The longer the time of observation, the nearer does this number approach to what we have called the probable number of bodies in the group.

The average number of bodies in a group depends on the limits which define the group, being, of course, greater when these limits are wide than when they are narrow. But it also depends on the character of the group, that is to say, the particular set of mean values of the conditions which entitle a body to be ranked in the group.

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[1] "Die kinetische Theorie der Gase in elementarer Darstellung, mit mathematischen Zusätzen. Von Dr. Oskar Emil Meyer, Professor de Physik an der Universität Breslau. (Breslau, 1877.)

It appears from the investigation that if ϕ be any property of a body, such that if ϕ_1 and ϕ_2 are its values or two bodies before an encounter, and Φ_1 and Φ_2 its values after the encounter, and if under all circumstances $\phi_1 + \phi_2 = \Phi_1 + \Phi_2$, and if the number of bodies in each group varies as $e^{-h\phi}$, then the distribution of the bodies in the groups will not be altered by the encounters between the bodies.

Now if we make ϕ equal to the sum of the kinetic and the potential energy of each body, the quantity $\phi_1 + \phi_2$ is not altered, either by an encounter between the two bodies or by external forces acting on them; so that a distribution according to the values of the function $e^{-h\phi}$ will satisfy the condition of permanence.

The most general case is that given in the seventh proposition. The bodies are no longer supposed to be smooth rigid-elastic spheres, but molecules, that is to say, material systems consisting of any number of parts acting on each other with forces of any kind consistent with the principle of the conservation of energy. The molecules of any one kind are supposed to have m degrees of freedom, this number being, in general, different in different kinds.

It is also assumed in the enunciation that all the forces in the system are either forces tending to fixed centres and functions of the distances from these centres, or else forces acting between the parts of the same molecule, thus excluding forces acting between one molecule and another except during the encounter of two molecules. This restriction, however, does not appear necessary, and indeed it is easy to remove it.

For the result of the proposition is to prove that if we define the group (A) of molecules as consisting of those whose generalised co-ordinates (q) are between certain limits (q and $q + dq$), and whose generalised momenta (p) are between certain other limits (p and $p + dp$), then the average number of molecules in the group is—

$$A e^{-h(\chi + T)} dp_1 \ldots dp_m\, dq_1 \ldots dq_m,$$

where A is a constant which is the same for all groups of molecules of the same kind, but is different for different kinds of molecules in the same mixture, but h is the same for all kinds of molecules. χ is the potential energy, and T the kinetic energy of a molecule when in the state (A), and $dp_1 \ldots dp_m$ are the differentials of the components of momentum, and $dq_1 \ldots dq_n$ the differentials of the co-ordinates. The continued product of these differentials specifies the extent of the group.

By integrating this expression with respect to any one of the variables, we may ascertain the average number of molecules in a larger group, in which that variable does not form a ground of subdivision. For instance, if we integrate with respect to all the co-ordinates, we arrive at a group consisting of all the molecules whose momenta are between certain limits, or by integrating with respect to the momenta we form a group of molecules whose configuration lies within certain limits.

In this way we obtain two very important results:—

1. The average kinetic energy of a molecule is $\frac{m}{2h}$ where m is the number of degrees of freedom of the molecule. This is independent of the position of the molecule.

2. The average number of molecules whose configuration lies between certain limits is—

$$A e^{-h\chi} dq_1 \ldots dq_n,$$

where χ is the potential energy of the molecule, arising from forces either internal to the molecule or tending to fixed centres, but (according to Mr. Watson) excluding intermolecular forces.

But as our definition of a molecule is of the most general kind, nothing is easier than to take into account any intermolecular forces by simply including within our "molecule" all those molecules between which intermolecular forces are exerted.

For instance, there is nothing to prevent us from defining as a molecule a material system consisting of one atom in Sirius, another in Arcturus, and a third in Aldebaran. If the universe is supposed to have attained that condition of thermal equilibrium to which alone these propositions apply, the average kinetic energy of each of these atoms will be $\frac{3}{2h}$, because each has three degrees of freedom.

That of the system of three atoms will be the sum of the kinetic energies of the three atoms, namely, $\frac{9}{2h}$. We might obtain the same result from the consideration that this system has nine degrees of freedom.

The centre of mass of the three atoms is a mathematical point at an immense distance from any of them. It has, of course, three degrees of freedom, and the kinetic energy of a material particle whose mass is the sum of the masses of the atoms and which moves as the centre of mass does is $\frac{3}{2h}$.

The value of the kinetic energy of the centre of mass will be the same for any system of atoms provided that every atom of the system is liable to encounters with atoms not belonging to the system. Of course if we take into our "molecule" all the atoms of a material system unconnected with any other system, its centre of mass will not be agitated at all by the mutual actions of the atoms during their encounters.

And here we must notice a point to which Mr. Watson has adverted only in a note at the foot of p. 20—the definition of the motion of the medium as distinguished from the motion of agitation of the molecules. For this is connected with the weak point of the demonstration and shows us the way to strengthen it.

The weak point of the demonstration is the tacit assumption that the sum of the potential and kinetic energies of a pair of molecules is the only function which does not change during their encounter. For there are other quantities which are not altered by the mutual action of two bodies such as their masses themselves, the sum of their momenta resolved in any given direction, and their angular momenta about any fixed axis.

Hence if instead of $T = \frac{1}{2} M (u^2 + v^2 + w^2)$ we write in the expression for the distribution of velocities—

$$\mathfrak{T} = T - M\ \{Uu + Vv + Ww + p\,(wy - vz) + q\,(uz - wx) + r\,(vx - uy) + CM,$$

the distribution of velocities will still be a permanent one.

In this expression the quantities U, V, W, p, q, r, may have any values provided they are the same for the whole system of bodies, but C may be different for different bodies, because it is multiplied by the mass of the body, which is invariable. But we arrive at the same expression by substituting in T for u, v, and w the quantities—

$$\begin{array}{l} u - U + qz - ry \\ v - V + rx - pz \\ w - W + py - qx; \end{array}$$

or, in other words, by substituting for the absolute velocities of the bodies their velocities relative to a system of axes moving in the most general manner possible; that is to say, the components of velocity of the origin being U, V, W, and the components of the velocity of rotation being p, q, r, and at the same adding to T the quantity

$$\tfrac{1}{2}\, M\left\{(qz - ry)^2 + (rx - pz)^2 + (py - qx)^2\right\} - C',$$

which depends on the co-ordinates only and not on the velocity of the body.

We now see that the most general case of permanent distribution is when the system of bodies is contained in a vessel of invariable form which moves with constant velocity along a screw, that is to say, in which one point is moving along a straight line with constant velocity, while the vessel rotates about an axis passing through this point with constant angular velocity.

When there is rotation, we must subtract from the potential energy a term depending on the co-ordinates, which shows that the rotation produces an effect similar to that of a centrifugal force at right angles to the axis of rotation.

Returning to the general expression for the number of molecules in a group, we may make it yield us information of other kinds. Thus, if we wish to know the density of a particular gas at any given point in the mixture, we have only to make the limits of the group those of an element of volume, and we find the density proportional to $e^{-h\chi}$, where χ is that part of the energy of a single molecule which is due to external forces, such as gravity. In the case of gravity, χ is equal to $m\,g\,z$, where m is the mass of a molecule, g the intensity of gravity, and z the height. This leads to the ordinary expression for the density of a gas of uniform temperature in a vertical column, and it shows that in the ultimate distribution of a mixture of gases the density of each gas diminishes with the height according to its own law, that is to say that of the heaviest gases diminishes most rapidly, so that the proportion of the heavier gases diminishes with the height.

This law of the distribution of gases was asserted by Dalton as a consequence of his theory of gases, and numerous experiments have been made on air collected at different heights in the atmosphere in order to detect a difference in their composition, but we cannot say that such a difference has as yet been satisfactorily established.

The atmosphere, in fact, is eminently unfitted for testing the theory of the ultimate state of a mixture in equilibrium, for the inequalities of temperature in so large a body of gas produce powerful currents which continually carry masses of the mixture from one stratum into another. This tends to produce a uniformity of composition and a variation of temperature which are both of them contrary to our theory of the condition of equilibrium, and which seem to favour certain other theories.

Nor is the case much improved if, instead of the open atmosphere, we substitute a mixture of gases contained in a vertical tube. For in order to obtain a difference of composition at the top and bottom of the tube large enough for experimental verification, the tube must be at least 100 metres high, and it would take more than a year for the contents of such a tube to approximate by one half to their final distribution. In the mean time the slightest difference of temperature in the sides of the tube would produce currents which would tend to equalise the composition of the mixture. To verify the other result of our theory—the uniformity of temperature in the ultimate state of a vertical column—would be attended with still greater difficulties.

But it would be quite within the powers of experimental methods to verify the law of distribution of a mixture of gases in a rotating vessel. Let two bulbs be connected by a wide tube, say 10 cm. long, and let them be filled with equal volumes of hydrogen and carbonic acid, well mixed together. Let this apparatus be placed on a whirling machine, so that one bulb shall be close to the axis, while the other is moving at the rate of fifty metres per second. The same degree of approximation to the final state, which would take years in the long tube, will be effected in minutes in this small apparatus, and the proportion of carbonic acid to hydrogen will be about $\frac{1}{140}$ greater in the bulb furthest from the axis.

The clear demonstration of this proposition given by Mr. Watson is of great scientific value, for almost every one of those who have attacked the question with insufficient methods of investigation have come to the conclusion that the temperature would diminish in a vertical column as the height increases; and those who regard gaseous diffusion from a chemical rather than a dynamical point of view would probably expect the composition to be uniform at all heights.

But the profound scientific value of this proposition becomes more manifest when we make use of it in establishing the definition of temperature and the law of volumes of gases.

In Prop. II. of this book, which corresponds to the original form of the theorem, as I gave it in the *Philosophical Magazine*, January, 1860, two sets of spheres are completely mixed up together in the same vessel, and it is proved that the average kinetic energy of a sphere is the same for either set. We may then assert, as I did, that the two gases are at the same temperature because they are thoroughly mixed together. But this assertion has no scientific meaning, because we cannot test its truth by putting a thermometer first into the one gas and then into the other.

But if we now call to our aid a system of forces acting on the molecules and tending to fixed centres, we may obtain a result capable of experimental verification; for though we are not acquainted with natural forces acting exclusively on one kind of gas, we can calculate the effects of such forces.

Let us assume, then, that the forces are such that the potential energy of a sphere of the set N is much greater in one part of a vessel, which we shall call *B*, than when it is in the part *A*, these two parts being separated by a stratum, *C*, within which the potential varies continuously. The medium consisting of the spheres *N* will be dense in *A*, it will become rarer in the stratum *C*, and there will be hardly any of these spheres in *B*.

Now let the potential energy of a sphere of the set *N′* be much greater when it is in *A* than when it is in *B*, and let it vary continuously from the one value to the other in the stratum *C*. Then the spheres of this set will be thickly scattered in *B*, will thin out in the stratum *C*, and will be very rare in *A*.

The two sets of spheres are thus kept in great measure separate in *A* and *B*, while free to exchange their kinetic energy by collisions within the stratum *C*.

Now by definition, the temperatures of two bodies are equal if, when the two bodies are placed in contact, their thermal state remains the same. We cannot apply this definition to the two sets of spheres in Prop. II., for they were inextricably mixed up together, but we have now got them almost completely separated from each other into two distinct regions. They are therefore practically distinct bodies, and we can test their temperatures separately.

Hence the statement, that the temperatures of two gases are equal when the kinetic energy of the centre of mass of a molecule is the same in each, is true, not only of gases mixed together, but of two pure gases in different parts of the same vessel.

If we assume that a powerful external force acts on each molecule tending to a fixed centre belonging to that molecule, each molecule will always remain very near its own fixed centre of force, and the assemblage of molecules will behave like a solid body. But forces of this kind are included among those considered in Prop. IV., so that the relation between temperature and the kinetic energy of the centre of mass of a single molecule must be extended even to solids.

Returning once more to the general expression for the average number of molecules in a group, we may make it yield us information with respect to the average number of sets of molecules which, at a given instant, are in a given configuration with respect to each other.

For instance, if two molecules act on each other, and if χ is the potential energy due to this action corresponding to a distance r, then the number of pairs of molecules whose distance is between r and $r + dr$ will be proportional to $r^2 e^{-h\chi}$. In the case of attraction, χ is negative, so that there will be a greater number of pairs of molecules within these limits of distance than there would have been if they did not attract each other. In the case of repulsion, χ is positive, so that the repulsion diminishes the number of pairs within the distance of repulsion. If the potential energy of a pair of molecules rapidly increases to an enormous value when the distance between their centres becomes less than a given quantity, the number of pairs which are within the given distance will be practically zero, and the molecules will behave like smooth rigid-elastic spheres.

By making the "molecule" include three or more molecules, and making χ the potential energy of this system, we may extend the theorem to the simultaneous encounter of three or more molecules, so that these cases, which were formally excluded in the earlier propositions, do not in any way interfere with the absolute generality of the final result.

The clear way in which Mr. Watson has demonstrated these propositions leaves us no escape from the terrible generality of his results. Some of these, no doubt, are very satisfactory to us in our present state of opinion about the constitution of bodies, but there are others which are likely to startle us out of our complacency, and perhaps ultimately to drive us out of all the hypotheses in which we have hitherto found refuge into that state of thoroughly conscious ignorance which is the prelude to every real advance in knowledge.

If we know from observation either the specific heat of a gas at constant pressure, or the ratio of its specific heats at constant pressure and at constant volume, we can determine the ratio of the rate of increase of its total energy to the rate of increase of the energy of agitation of the centres of its molecules. Now if the molecule has *m* degrees of freedom, its total kinetic energy is to the energy of agitation of its centre of mass as *m* to 3. It is probable that the internal potential energy of the molecule increases as the temperature rises, and this would make the ratio of the whole energy to that of agitation of centres greater than that of *m* to 3, so that if we know this ratio by experiment, we can assert that *m* cannot exceed a certain value.

For chlorine, ammonia, and sulphuretted hydrogen, *m* cannot exceed 6; for hydrogen, oxygen, nitrogen, air, carbonic oxide, nitrous oxide, and hydrochloric acid, it cannot exceed 5, and for mercury gas, according to the experiments of Kundt and Warburg, it cannot exceed 3.

Now Boltzmann has pointed out in a paper: "Über die Natur der Gasmolecüle" (Vienna Acad., December 14, 1876), that if the molecules were rigid-elastic bodies of any form, *m* would be 6, that if they were smooth figures of revolution, the velocity of rotation about the axis of figure would not be affected by the collisions, so that *m* would be 5, and that if they were smooth spheres, the three component velocities of rotation would each of them be independent of collisions, so that *m* would be reduced to 3, and these values are in striking agreement with the phenomena of the three groups of gases.

But before we accept this somewhat promising hypothesis, let us try to construct a rigid-elastic body. It will not do to take a body formed of continuous matter endowed with elastic properties, and to increase the coefficients of elasticity without limit till the body becomes practically rigid. For such a body, though apparently rigid, is in reality capable of internal vibrations, and these of an infinite variety of types, so that the body has an infinite number of degrees of freedom.

The same objection applies to all atoms constructed of continuous, non-rigid matter, such as the vortex-atoms of Thomson. Such atoms would soon convert all their energy of agitation into internal energy, and the specific heat of a substance composed of them would be infinite.

A truly rigid-elastic body is one whose encounters with similar bodies take place as if both were elastic, but which is not capable of being set into a state of internal vibration. We must take a perfectly rigid body and endow it with the power of repelling all other bodies, but only when they come within a very short distance from its surface, but then so strongly that under no circumstances whatever can any body come into actual contact with it.

This appears to be the only constitution we can imagine for a rigid-elastic body. And now that we have got it, the best thing we can do is to get rid of the rigid nucleus altogether, and substitute for it an atom of Boscovich—a mathematical point endowed with mass and with powers of acting at a distance on other atoms.

But Boltzmann's molecules are not absolutely rigid. He admits that they vibrate after collisions, and that their vibrations are of several different types, as the spectroscope tells us. But still he tries to make us believe that these vibrations are of small importance as regards the principal part of the motion of the molecules. He compares them to billiard balls, which, when they strike each other, vibrate for a short time, but soon give up the energy of their vibration to the air, which carries far and wide the sound of the click of the balls.

In like manner, the light emitted by the molecules shows that their internal vibrations after each collision are quickly given up to the luminiferous ether.

If we were to suppose that at ordinary temperatures the collisions are not severe enough to produce any internal vibrations, and that these occur only at temperatures like that of the electric spark, at which we cannot make measurements of specific heat, we might, perhaps, reconcile the spectroscopic results with what we know about specific heat.

But the fixed position of the bright lines of a gas shows that the vibrations are isochronous, and therefore that the forces which they call into play vary directly as the relative displacements, and if this be the character of the forces, all impacts, however slight, will produce vibrations.

Besides this, even at ordinary temperatures, in certain gases, such as iodine gas and nitrous acid, absorption bands exist, which indicate that the molecules are set into internal vibration by the incident light.

The molecules, therefore, are capable, as Boltzmann points out, of exchanging energy with the ether.

But we cannot force the ether into the service of our theory so as to take from the molecules their energy of internal vibration and give it back to them as energy of translation. It cannot in any way interfere with the ratio between these two kinds of energy which Boltzmann himself has established. All it can do is to take up its own due proportion of energy according to the number of its degrees of freedom.

We leave it to the authors of the "Unseen Universe" to follow out the consequences of this statement.

J. CLERK MAXWELL

OUR BOOK SHELF

Report on the Progress and Condition of the Royal Gardens at Kew during the Year 1876. (Clowes and Sons.)

SIR JOSEPH HOOKER'S annual report on the Royal Gardens, Kew, for 1876, has just been issued. It is a pamphlet of some thirty-three pages, and is a considerable improvement on the reports of former years. It deals most fully with new plants of economic interest, whether such have been actually received or sent from the Royal Gardens, or have formed the subject of correspondence with foreign or colonial governments. It is eminently satisfactory to know that such useful plants as the Para rubber (*Hevea brasiliensis*), the ipecacuanha (*Cephælis ipecacuanha*), the Liberian coffee (*Coffea liberica*), and others, have been most successfully introduced into India and other countries, through the instrumentality of Kew. Of the 70,000 seeds of the *Hevea* received at Kew about the middle of June last year, all of which we are told were at once sown, and though closely packed together, covered a space of over 300 square feet so soon as August 12th following, upwards of 1,900 living plants, raised from these seeds, were transmitted to Ceylon in thirty-eight Wardian cases, 90 per cent. of the whole consignment reaching Dr. Thwaites in excellent condition. So rapid was the germination of these seeds at Kew that some had actually started into growth on the fourth day after sowing, and many in a few days reached a height of eighteen inches. It has been arranged that these young plants shall "be nursed and established in Ceylon for subsequent transmission through the Indian Gardens to Assam, Burma, and other hot damp provinces of India proper." Besides those sent to India, smaller quantities of plants have also been despatched to the west coast of Africa, Burma, Dominica, Jamaica, Java, Queensland, Singapore, and Trinidad. With regard to ipecacuanha, though Dr. King reports that he fears it cannot be grown so far north in India as Bengal, it is nevertheless in some situations capable of rapid and extensive cultivation, and the roots grown in India have been proved to be quite as efficacious in a medicinal point of view as those from the best districts of South America. In the matter of Liberian coffee, the wide and general extension of this new kind in coffee-growing countries bids fair, in many parts, to entirely supersede the old and better known *Coffea arabica*. Sir Joseph Hooker reports the receipt of numerous favourable notices of the plant, and quotes "two from opposite sides of the world," namely Ceylon and Dominica. With reference to diseases affecting coffee plants—which it is hoped the more sturdy habit of the Liberian kind will help it in some measure to resist—a very exhaustive notice is furnished, which is not only of much interest in a scientific point of view, but cannot fail to be valuable to coffee-planters themselves. It will, moreover, no doubt be the means of causing more careful observations to be made by residents on coffee estates or in coffee-growing countries into the nature and habits of diseases which are still more or less obscure.

Considerable additions are reported to the Museums and Herbarium, the new building for the accommodation of the latter collection being now in a very advanced state. The new Laboratory, which has been erected at the expense of T. J. Phillips Joddrell, Esq., is reported as having been completed during the year, and though not fully provided with the necessary equipment at the time the report was written, has been already, as our readers are aware, used by Dr. Tyndall in several of his recent experiments and researches.

Two new features of the report which we have not already mentioned are—first, the introduction of plates, one being a figure of the new Liberian coffee plant, and the other a view and ground-plan of the Laboratory; and second, the publication of the report, at a charge of sixpence, by Messrs. Clowes and Sons.

Natural History Transactions of Northumberland and Durham, vol. v., part 3. (Williams and Norgate, 1877.)

THIS part is by no means the least valuable of these transactions; on the contrary, it will rank high, owing to the contributions of Dr. Embleton and Mr. Atthey on the structure of the Labyrinthodonts, and the eight excellent plates by which their papers are illustrated. The illustrations of Loxomma and Anthracosaurus are as complete and instructive as any that have yet been pub-